College of Engineering
Dept. of the electrical power and Machine
Subject/ ANALOG ELECTRONIC

stage / second
Date: Mon24/11/2014
Time/ 3 hours

BJT Small-Signal Analysis
Common –Emitter Fixed-Bias Configuration

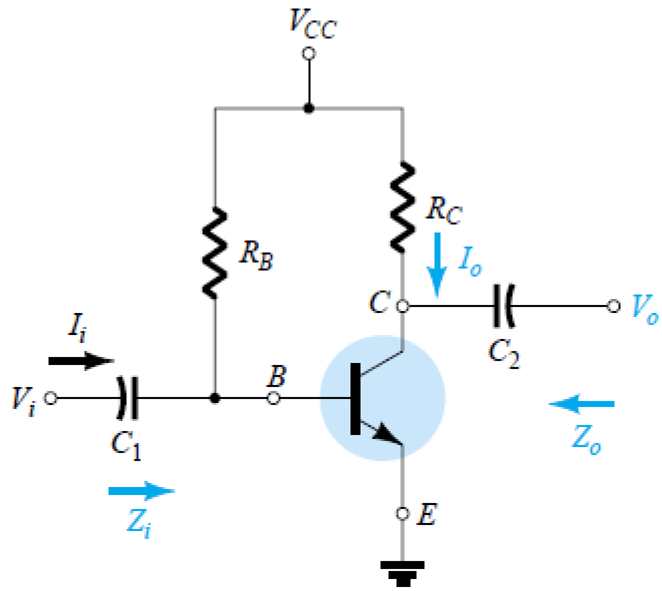

Fig.1: Common-emitter fixed-bias configuration.

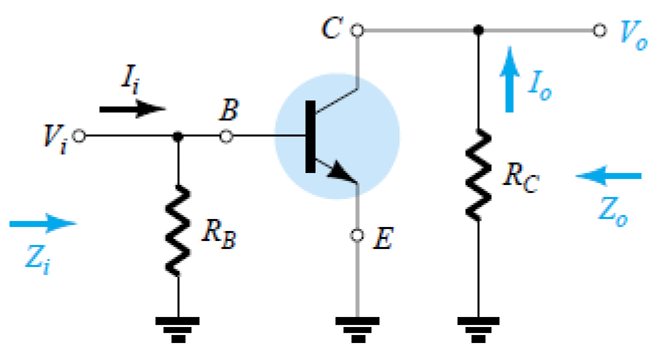

Fig.2: Network of Fig.1 following the removal of the effects of V_{CC}, C_1, and C_2.

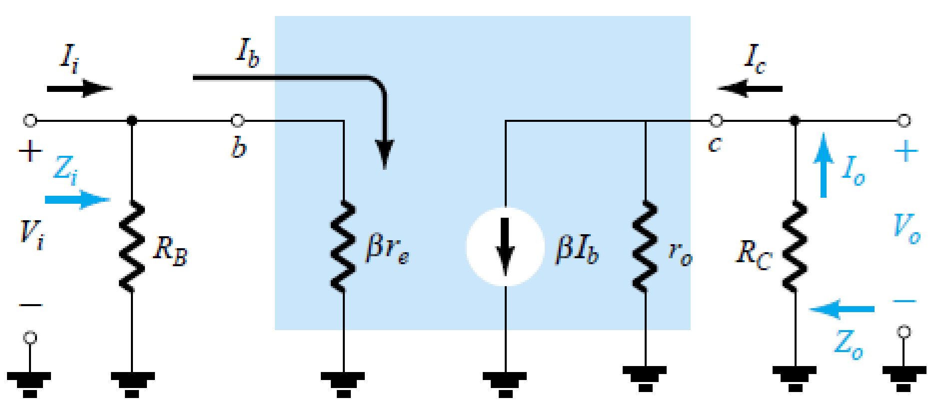

Fig.3: Substituting the re model into the network of Fig.2

$$\boxed{Z_i = R_B \| \beta r_e} \quad \text{ohms}$$

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$$\boxed{Z_i \cong \beta r_e}_{R_B \geq 10\beta r_e} \quad \text{ohms}$$

***Zo*:** Recall that the output impedance of any system is defined as the impedance *Zo* determined when *Vi* = 0. For Fig.3, when *Vi* = 0, *Ii* = *Ib* = 0, resulting in an open-circuit equivalence for the current source. The result is the configuration of Fig.4.

$$\boxed{Z_o = R_C \| r_o} \quad \text{ohms}$$

If *ro* ≥10 *Rc*, the approximation *RC* // *ro* = *RC* is frequently applied and

$$\boxed{Z_o \cong R_C}_{r_o \geq 10R_C}$$

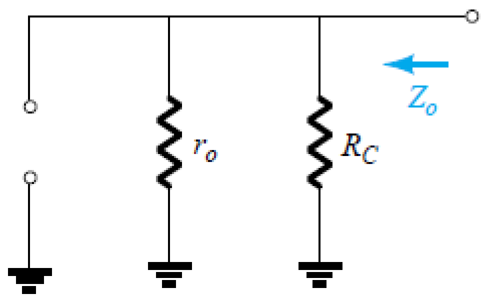

Fig.4 Determining *Zo*

***A*$_v$:** The resistors r_o and R_C are in parallel,

and $$V_o = -\beta I_b(R_C \| r_o)$$

but $$I_b = \frac{V_i}{\beta r_e}$$

so that $$V_o = -\beta\left(\frac{V_i}{\beta_{r_e}}\right)(R_C \| r_o)$$

and $$\boxed{A_v = \frac{V_o}{V_i} = -\frac{(R_C \| r_o)}{r_e}}$$

If $r_o \geq 10R_C$,

$$\boxed{A_v = -\frac{R_C}{r_e}}_{r_o \geq 10R_C}$$

***A*$_i$:** The current gain is determined in the following manner: Applying the current-divider rule to the input and output circuits,

$$I_o = \frac{(r_o)(\beta I_b)}{r_o + R_C} \quad \text{and} \quad \frac{I_o}{I_b} = \frac{r_o \beta}{r_o + R_C}$$

with $$I_b = \frac{(R_B)(I_i)}{R_B + \beta r_e} \quad \text{or} \quad \frac{I_b}{I_i} = \frac{R_B}{R_B + \beta r_e}$$

The result is

$$A_i = \frac{I_o}{I_i} = \left(\frac{I_o}{I_b}\right)\left(\frac{I_b}{I_i}\right) = \left(\frac{r_o\beta}{r_o + R_C}\right)\left(\frac{R_B}{R_B + \beta r_e}\right)$$

and

$$\boxed{A_i = \frac{I_o}{I_i} = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)}}$$

which is certainly an unwieldy, complex expression.

However, if $r_o \geq 10R_C$ and $R_B \geq 10\beta r_e$, which is often the case,

$$A_i = \frac{I_o}{I_i} \cong \frac{\beta R_B r_o}{(r_o)(R_B)}$$

and

$$\boxed{A_i \cong \beta}_{r_o \geq 10R_C,\ R_B \geq 10\beta r_e}$$

Recall from chapter 7 we can use equation below to avoid complexity

$$\boxed{A_i = -A_v \frac{Z_i}{R_C}}$$

Phase Relationship: The negative sign in the resulting equation for *Av* reveals that a 180° phase shift occurs between the input and output signals

For the network of Fig. 5 :

(a) Determine r_e.
(b) Find Z_i (with $r_o = \infty\ \Omega$).
(c) Calculate Z_o (with $r_o = \infty\ \Omega$).
(d) Determine A_v (with $r_o = \infty\ \Omega$).
(e) Find A_i (with $r_o = \infty\ \Omega$).
(f) Repeat parts (c) through (e) including $r_o = 50\ \text{k}\Omega$ in all calculations and compare results.

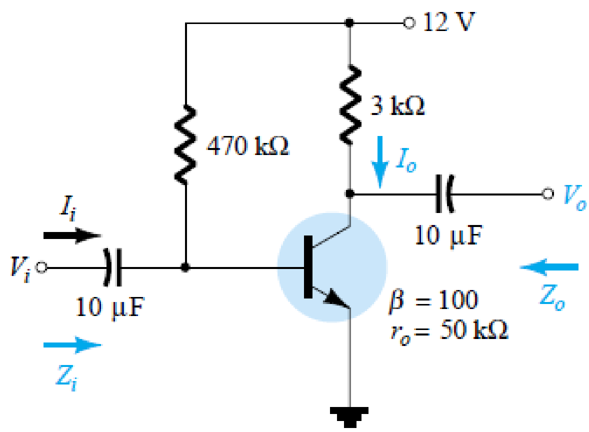

Figure 5 Example 8.1.

Solution

(a) DC analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega} = 24.04\ \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (101)(24.04\ \mu\text{A}) = 2.428\text{ mA}$$

$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{2.428\text{ mA}} = \mathbf{10.71\ \Omega}$$

(b) $\beta r_e = (100)(10.71\ \Omega) = 1.071\text{ k}\Omega$

$Z_i = R_B \| \beta r_e = 470\text{ k}\Omega \| 1.071\text{ k}\Omega = \mathbf{1.069\text{ k}\Omega}$

(c) $Z_o = R_C = \mathbf{3\text{ k}\Omega}$

(d) $A_v = -\dfrac{R_C}{r_e} = -\dfrac{3\text{ k}\Omega}{10.71\ \Omega} = \mathbf{-280.11}$

(e) Since $R_B \geq 10\beta r_e (470\text{ k}\Omega > 10.71\text{ k}\Omega)$

$A_i \cong \beta = \mathbf{100}$

(f) $Z_o = r_o \| R_C = 50\ \text{k}\Omega \| 3\ \text{k}\Omega = \mathbf{2.83\ k\Omega}$ vs. $3\ \text{k}\Omega$

$$A_v = -\frac{r_o \| R_C}{r_e} = \frac{2.83\ \text{k}\Omega}{10.71\ \Omega} = \mathbf{-264.24} \text{ vs. } -280.11$$

$$A_i = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)} = \frac{(100)(470\ \text{k}\Omega)(50\ \text{k}\Omega)}{(50\ \text{k}\Omega + 3\ \text{k}\Omega)(470\ \text{k}\Omega + 1.071\ \text{k}\Omega)}$$
$$= \mathbf{94.13} \text{ vs. } 100$$

As a check:

$$A_i = -A_v \frac{Z_i}{R_C} = \frac{-(-264.24)(1.069\ \text{k}\Omega)}{3\ \text{k}\Omega} = \mathbf{94.16}$$

which differs slightly only due to the accuracy carried through the calculations.

VOLTAGE-DIVIDER BIAS

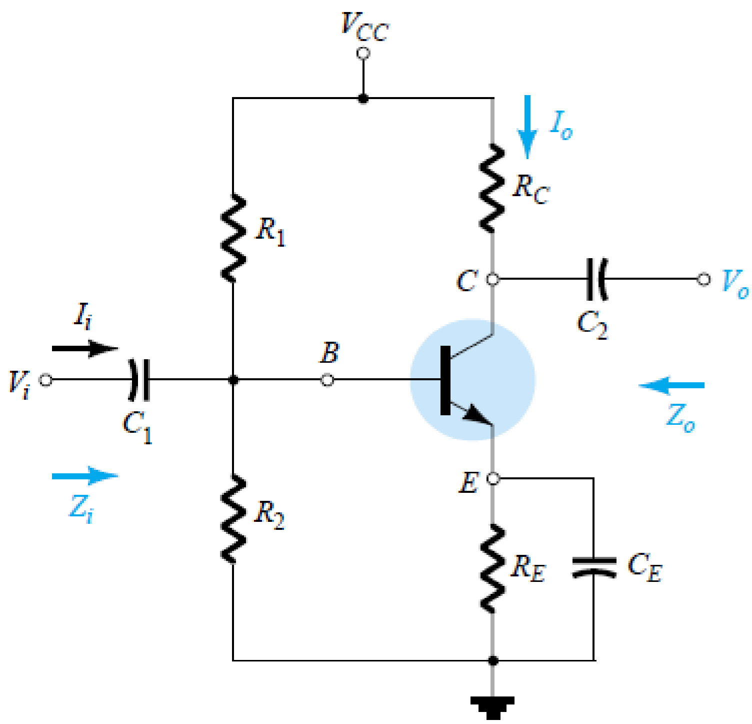

Fig.6: Voltage-divider bias configuration.

Substituting the *re* equivalent circuit will result in the network of Fig.7. Note the absence of R_E due to the low-impedance shorting effect of the bypass capacitor, C_E. That is, at the frequency (or frequencies) of operation, the reactance of the capacitor is so small compared to R_E that it is treated as a short circuit across R_E.

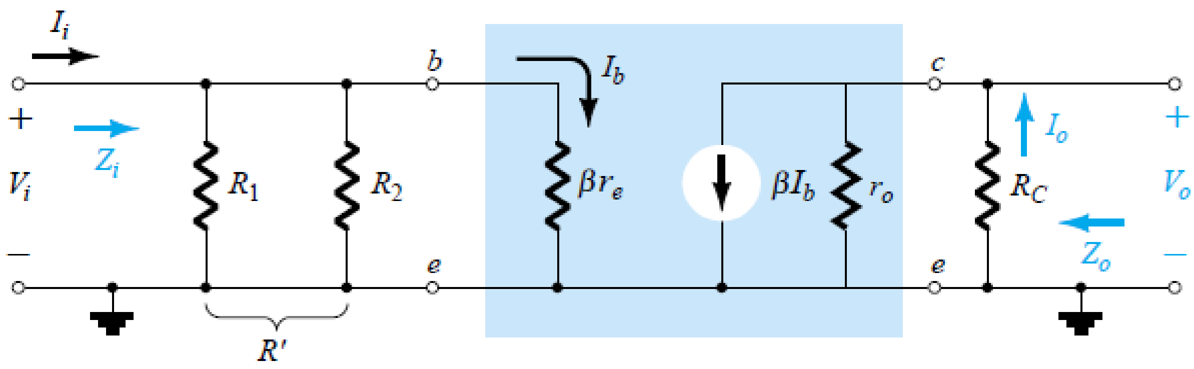

Fig.7: Substituting the *re* equivalent circuit into the ac equivalent network of Fig.6

When V_{CC} is set to zero, it places one end of R_1 and R_C at ground potential as shown in Fig.7. In addition, note that R_1 and R_2 remain part of the input circuit while R_C is part of the output circuit. The parallel combination of R_1 and R_2 is defined by:

$$\boxed{R' = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}}$$

From Fig.7 Zi is

$$\boxed{Z_i = R' \| \beta r_e}$$

Zo: From Fig.7 with *Vi* set to 0 V resulting in $I_b = 0$ μA and $\beta I_b = 0$ mA,

$$\boxed{Z_o = R_C \| r_o}$$

If $r_o \geq 10R_C$,

$$\boxed{Z_o \cong R_C}_{r_o \geq 10R_C}$$

A_v: Since R_C and r_o are in parallel,

$$V_o = -(\beta I_b)(R_C \| r_o)$$

and

$$I_b = \frac{V_i}{\beta r_e}$$

so that

$$V_o = -\beta \left(\frac{V_i}{\beta r_e}\right)(R_C \| r_o)$$

and

$$\boxed{A_v = \frac{V_o}{V_i} = \frac{-R_C \| r_o}{r_e}}$$

***Ai*:** Since the network of Fig.7 is so similar to that of Fig.3 except for the fact that $R' = R1\backslash\backslash R2 = RB$. That is,

$$\boxed{A_i = \frac{I_o}{I_i} = \frac{\beta R' r_o}{(r_o + R_C)(R' + \beta r_e)}}$$

For $r_o \geq 10R_C$,

$$A_i = \frac{I_o}{I_i} \cong \frac{\beta R' r_o}{r_o(R' + \beta r_e)}$$

and

$$\boxed{A_i = \frac{I_o}{I_i} \cong \frac{\beta R'}{R' + \beta r_e}}_{r_o \geq 10R_C}$$

And if $R' \geq 10\beta r_e$,

$$A_i = \frac{I_o}{I_i} = \frac{\beta R'}{R'}$$

and

$$\boxed{A_i = \frac{I_o}{I_i} \cong \beta}_{r_o \geq 10R_C,\ R' \geq 10\beta r_e}$$

As an option,

$$\boxed{A_i = -A_v \frac{Z_i}{R_C}}$$

EXAMPLE 8.2 For the network of Fig. 8 , determine:

(a) r_e.
(b) Z_i.
(c) Z_o ($r_o = \infty\ \Omega$).
(d) A_v ($r_o = \infty\ \Omega$).
(e) A_i ($r_o = \infty\ \Omega$).
(f) The parameters of parts (b) through (e) if $r_o = 1/h_{oe} = 50\text{ k}\Omega$ and compare results.

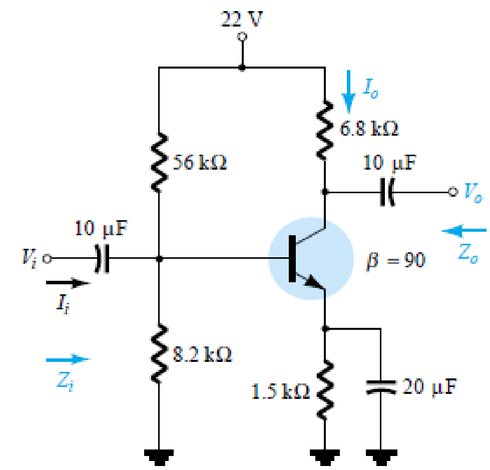

Figure 8. Example 8.2.

Solution

(a) DC: Testing $\beta R_E > 10R_2$

$$(90)(1.5\text{ k}\Omega) > 10(8.2\text{ k}\Omega)$$
$$135\text{ k}\Omega > 82\text{ k}\Omega\ (\textit{satisfied})$$

Using the approximate approach,

$$V_B = \frac{R_2}{R_1 + R_2}V_{CC} = \frac{(8.2\text{ k}\Omega)(22\text{ V})}{56\text{ k}\Omega + 8.2\text{ k}\Omega} = 2.81\text{ V}$$
$$V_E = V_B - V_{BE} = 2.81\text{ V} - 0.7\text{ V} = 2.11\text{ V}$$

$$I_E = \frac{V_E}{R_E} = \frac{2.11\text{ V}}{1.5\text{ k}\Omega} = 1.41\text{ mA}$$

$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{1.41\text{ mA}} = \mathbf{18.44\ \Omega}$$

(b) $R' = R_1 \| R_2 = (56\text{ k}\Omega)\|(8.2\text{ k}\Omega) = 7.15\text{ k}\Omega$

$$Z_i = R'\|\beta r_e = 7.15\text{ k}\Omega\|(90)(18.44\ \Omega) = 7.15\text{ k}\Omega\|1.66\text{ k}\Omega$$
$$= \mathbf{1.35\text{ k}\Omega}$$

(c) $Zo = RC =$ **6.8 kΩ**

(d) $A_v = -\dfrac{R_C}{r_e} = -\dfrac{6.8\text{ k}\Omega}{18.44\ \Omega} = \mathbf{-368.76}$

(e) The condition $R' \geq 10\beta r_e$ (7.15 kΩ ≥ 10(1.66 kΩ) = 16.6 kΩ is *not* satisfied. Therefore,

$$A_i \cong \frac{\beta R'}{R' + \beta r_e} = \frac{(90)(7.15\text{ k}\Omega)}{7.15\text{ k}\Omega + 1.66\text{ k}\Omega} = \mathbf{73.04}$$

(f) $Z_i = \mathbf{1.35\ k\Omega}$

$Z_o = R_C \| r_o = 6.8\text{ k}\Omega \| 50\text{ k}\Omega = \mathbf{5.98\ k\Omega}$ vs. 6.8 kΩ

$A_v = -\dfrac{R_C \| r_o}{r_e} = -\dfrac{5.98\text{ k}\Omega}{18.44\ \Omega} = \mathbf{-324.3}$ vs. −368.76

The condition

$$r_o \geq 10R_C\ (50\text{ k}\Omega \geq 10(6.8\text{ k}\Omega) = 68\text{ k}\Omega)$$

is *not* satisfied. Therefore,

$$A_i = \frac{\beta R' r_o}{(r_o + R_C)(R' + \beta r_e)} = \frac{(90)(7.15\text{ k}\Omega)(50\text{ k}\Omega)}{(50\text{ k}\Omega + 6.8\text{ k}\Omega)(7.15\text{ k}\Omega + 1.66\text{ k}\Omega)}$$
$$= \mathbf{64.3} \text{ vs. } 73.04$$

There was a measurable difference in the results for Z_o, A_v, and A_i because the condition $r_o \geq 10R_C$ was *not* satisfied.

CE EMITTER-BIAS CONFIGURATION

Unbypassed

The *re* equivalent model is substituted in Fig. 10, but note the absence of the resistance *ro*. The effect of *ro* is to make the analysis a great deal more complicated.

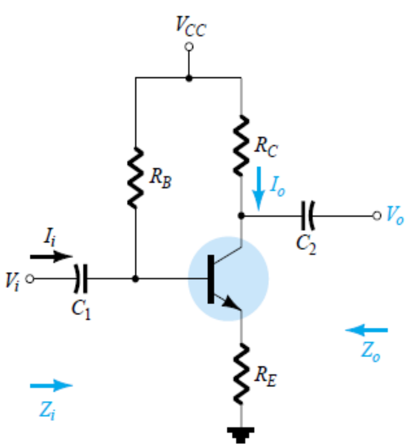

Fig.9: CE emitter-bias configuration.

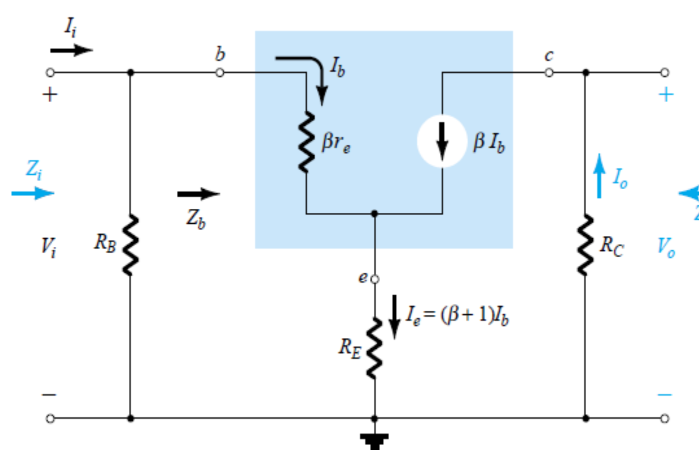

Fig.10: Substituting the *re* equivalent circuit into the ac equivalent network of Fig.9.

Applying Kirchhoff's voltage law to the input side of Fig.9 will result in:

$$V_i = I_b\beta r_e + I_e R_E$$

$$V_i = I_b\beta r_e + (\beta + 1)I_b R_E$$

And the input impedance looking into the network to the right of *RB is*

$$Z_b = \frac{V_i}{I_b} = \beta r_e + (\beta + 1)R_E$$

The result as displayed in Fig.11 reveals that the input impedance of a transistor with an unbypassed resistor *RE* is determined by

$$\boxed{Z_b = \beta r_e + (\beta + 1)R_E}$$

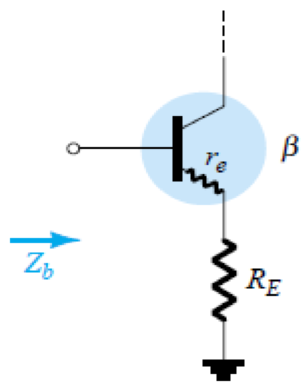

Fig.11: Defining the input impedance of a transistor with an unbypassed emitter resistor.

Since β is normally much greater than 1, the approximate equation is the following:

$$Z_b \cong \beta r_e + \beta R_E$$

and

$$\boxed{Z_b \cong \beta(r_e + R_E)}$$

Since *RE* is often much greater than *re*, can be further reduced to

$$\boxed{Z_b \cong \beta R_E}$$

Zi: Returning to Fig.10, we have

$$\boxed{Z_i = R_B \| Z_b}$$

Zo: With *Vi* set to zero, *Ib* = 0 and β*Ib* can be replaced by an open-circuit equivalent. The result is:

$$\boxed{Z_o = R_C}$$

A_v:

$$I_b = \frac{V_i}{Z_b}$$

and

$$V_o = -I_o R_C = -\beta I_b R_C$$

$$= -\beta\left(\frac{V_i}{Z_b}\right) R_C$$

with

$$\boxed{A_v = \frac{V_o}{V_i} = -\frac{\beta R_C}{Z_b}}$$

Substituting $Z_b = \beta(r_e + R_E)$ gives

$$\boxed{A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_e + R_E}}$$

and for the approximation $Z_b \cong \beta R_E$,

$$\boxed{A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{R_E}}$$

Note again the absence of β from the equation for *Av*.

Ai: The magnitude of R_B is often too close to Z_b to permit the approximation $I_b = I_i$. Applying the current-divider rule to the input circuit will result in

$$I_b = \frac{R_B I_i}{R_B + Z_b}$$

and

$$\frac{I_b}{I_i} = \frac{R_B}{R_B + Z_b}$$

In addition,

$$I_o = \beta I_b$$

and

$$\frac{I_o}{I_b} = \beta$$

so that

$$A_i = \frac{I_o}{I_i} = \frac{I_o}{I_b}\frac{I_b}{I_i}$$

$$= \beta \frac{R_B}{R_B + Z_b}$$

and

$$\boxed{A_i = \frac{I_o}{I_i} = \frac{\beta R_B}{R_B + Z_b}}$$

or

$$A_i = -A_v \frac{Z_i}{R_C}$$

Phase relationship: The negative sign in Eq.again reveals a 180° phase shift between *Vo* and *Vi*.

Bypassed

If *RE* of Fig.9 is bypassed by an emitter capacitor *CE*, the complete *re* equivalent model can be substituted resulting in the same equivalent network as Fig.3 so all Equations are therefore applicable.

EXAMPLE 8.3: For the network of Fig.12, without C_E (unbypassed), determine:

(a) r_e.
(b) Z_i.
(c) Z_o.
(d) A_v.
(e) A_i.

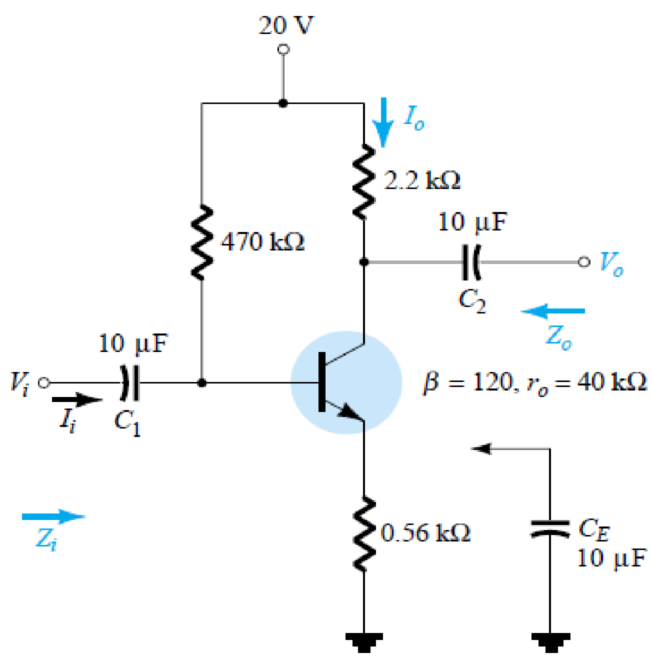

Fig.12: for EXAMPLE 8.3

Solution

(a) DC: $$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} = \frac{20\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega + (121)0.56\text{ k}\Omega} = 35.89\ \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (121)(46.5\ \mu\text{A}) = 4.34\text{ mA}$$

and $$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{4.34\text{ mA}} = \mathbf{5.99\ \Omega}$$

(b) Testing the condition $r_o \geq 10(R_C + R_E)$,

$$40 \text{ k}\Omega \geq 10(2.2 \text{ k}\Omega + 0.56 \text{ k}\Omega)$$
$$40 \text{ k}\Omega \geq 10(2.76 \text{ k}\Omega) = 27.6 \text{ k}\Omega \ (\textit{satisfied})$$

Therefore,

$$Z_b \cong \beta(r_e + R_E) = 120(5.99\ \Omega + 560\ \Omega) = 67.92 \text{ k}\Omega$$

and

$$Z_i = R_B \| Z_b = 470 \text{ k}\Omega \| 67.92 \text{ k}\Omega = \mathbf{59.34 \text{ k}\Omega}$$

(c) $Z_o = R_C = \mathbf{2.2 \text{ k}\Omega}$

(d) $r_o \geq 10R_C$ is satisfied. Therefore,

$$A_v = \frac{V_o}{V_i} \cong -\frac{\beta R_C}{Z_b} = -\frac{(120)(2.2 \text{ k}\Omega)}{67.92 \text{ k}\Omega} = \mathbf{-3.89}$$

compared to -3.93 using Eq. $A_v = \dfrac{V_o}{V_i} \cong -\dfrac{R_C}{R_E}$.

(e) $A_i = -A_v \dfrac{Z_i}{R_C} = -(-3.89)\left(\dfrac{59.34 \text{ k}\Omega}{2.2 \text{ k}\Omega}\right) = \mathbf{104.92}$

compared to 104.85 using Eq. $A_i = \dfrac{I_o}{I_i} = \dfrac{\beta R_B}{R_B + Z_b}$

EXAMPLE 8.4

Repeat the analysis of Example 8.3 with C_E in place.

Solution

(a) The dc analysis is the same, and $r_e = 5.99\ \Omega$.

(b) R_E is "shorted out" by C_E for the ac analysis. Therefore,

$$Z_i = R_B \| Z_b = R_B \| \beta r_e = 470 \text{ k}\Omega \| (120)(5.99\ \Omega) = 470 \text{ k}\Omega \| 718.8\ \Omega \cong \mathbf{717.70\ \Omega}$$

(c) $Z_o = R_C = \mathbf{2.2 \text{ k}\Omega}$

(d) $A_v = -\dfrac{R_C}{r_e}$

$$= -\frac{2.2 \text{ k}\Omega}{5.99\ \Omega} = \mathbf{-367.28} \quad \text{(a significant increase)}$$

(e) $A_i = \dfrac{\beta R_B}{R_B + Z_b} = \dfrac{(120)(470 \text{ k}\Omega)}{470 \text{ k}\Omega + 718.8\ \Omega} = \mathbf{119.82}$

EXAMPLE 8.5 For the network of Fig. 13, determine (using appropriate approximations):

(a) r_e.
(b) Z_i.
(c) Z_o.
(d) A_v.
(e) A_i.

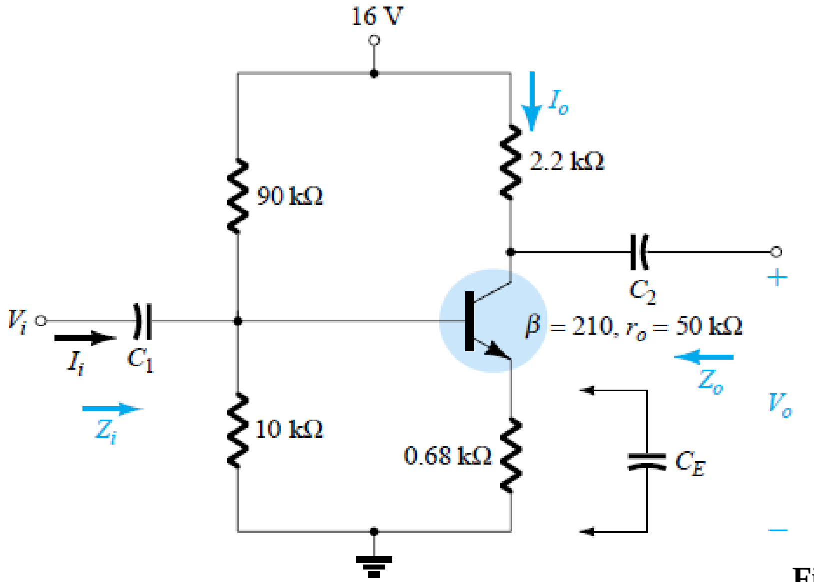

Fig.13

Solution

(a) Testing $\beta R_E > 10R_2$

$$(210)(0.68\ \text{k}\Omega) > 10(10\ \text{k}\Omega)$$

$$142.8\ \text{k}\Omega > 100\ \text{k}\Omega\ (\textit{satisfied})$$

$$V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{10\ \text{k}\Omega}{90\ \text{k}\Omega + 10\ \text{k}\Omega}(16\ \text{V}) = 1.6\ \text{V}$$

$$V_E = V_B - V_{BE} = 1.6\ \text{V} - 0.7\ \text{V} = 0.9\ \text{V}$$

$$I_E = \frac{V_E}{R_E} = \frac{0.9\ \text{V}}{0.68\ \text{k}\Omega} = 1.324\ \text{mA}$$

$$r_e = \frac{26\ \text{mV}}{I_E} = \frac{26\ \text{mV}}{1.324\ \text{mA}} = \mathbf{19.64\ \Omega}$$

(b) The ac equivalent circuit is provided in Fig.14. The resulting configuration is now different from Fig.10 only by the fact that now

$$R_B = R' = R_1 \| R_2 = 9\ \text{k}\Omega$$

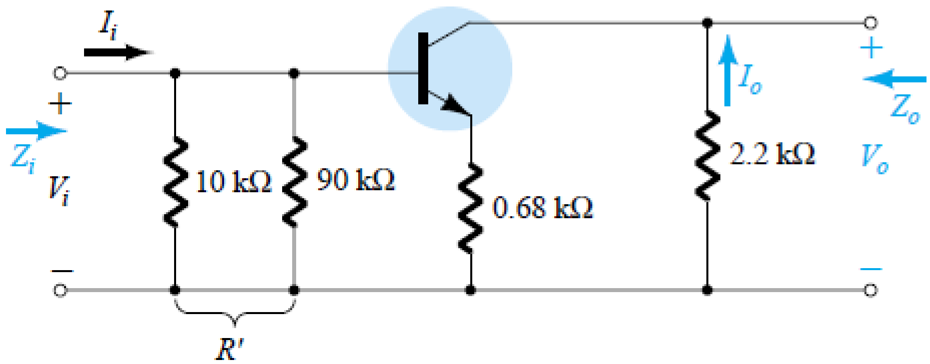

Fig.14: The ac equivalent circuit of Fig.13.

The testing conditions of $r_o \geq 10(R_C + R_E)$ and $r_o \geq 10R_C$ are both satisfied. Using the appropriate approximations yields

$$Z_b \cong \beta R_E = 142.8\ \text{k}\Omega$$

$$Z_i = R_B \| Z_b = 9\ \text{k}\Omega \| 142.8\ \text{k}\Omega$$
$$= \mathbf{8.47\ k\Omega}$$

(c) $Z_o = R_C = \mathbf{2.2\ k\Omega}$

(d) $A_v = -\dfrac{R_C}{R_E} = -\dfrac{2.2\ \text{k}\Omega}{0.68\ \text{k}\Omega} = \mathbf{-3.24}$

(e) $A_i = -A_v \dfrac{Z_i}{R_C} = -(-3.24)\left(\dfrac{8.47\ \text{k}\Omega}{2.2\ \text{k}\Omega}\right)$
$= \mathbf{12.47}$